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COMMENT

On the two-parameter theory of solitons in spin systems

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Abstract. We demonstrate the consistency of the method that introduces two parameters, the inverse of the square root of the spin length and the characteristic soliton length, in the study of solitons in spin systems.

Non-linear phenomena in spin systems have been a subject of interest for nearly 20 years [1]. Non-linear equations of motion were first studied in detail for classical systems [2]. In a quantum spin system, a bosonic representation of the spin operators turns out to be a very suitable method for studying the solitary waves, because they allow one to include quantum corrections in a systematic way. A relevant standard approach is that the non-linear Schrödinger equation was obtained from the expansion of the Holstein–Primakoff (HP) transformation [3]. Working in the coherent state representation of Glauber [4], and making a long-wave approximation, one then finds solitary wave profiles from analogy with classical solitons, which is the so called semiclassical treatment [5].

These studies on soliton excitations in a one-dimensional Heisenberg ferromagnet in the HP representation of spin operators are based on two approximations: one is the semiclassical approximation which takes $1/\sqrt{S}$ (S is the spin length) as the small parameter and the other is the long-wave limit in which the coherent amplitudes are expanded to a^2 terms (a is the lattice constant). The two approximations are thought to be independent of each other [5]. There was an interesting arguement among these articles [5] as to whose non-linear Schrödinger equation was more suitable within the framework of the HP representation.

In fact, for the coexistence of two perturbations, the relative ratio of two small parameters plays an important role in obtaining the proper non-linear wave equations. Then we developed a two-parameter theory of solitons in spin systems [6–9] to take it into account. Recently, based on an improper normalization of the eigenvalue of the coherent state, Kapor *et al* [10] made a misleading statement that the two-parameter theory is inconsistent. In this comment, we show that their demonstration is not valid and the two-parameter theory of solitons is not inconsistent.

Firstly, we briefly review our approach. One can introduce the HP representation for spin operators:

$$\begin{aligned} S_i^z &= \hbar(S - a_i^+ a_i) \\ S_i^+ &= \hbar(2S - a_i^+ a_i)^{1/2} a_i \\ S_i^- &= (S_i^+)^+ \end{aligned} \qquad (1)$$

where a_i^+ and a_i are Bose operators. One can use the expansion in terms of the dimensionless parameter $\varepsilon = 1/\sqrt{S}$ to obtain

$$\begin{aligned} S_i^+ &= \sqrt{2}S\hbar[\varepsilon a_i - \varepsilon^3 a_i^+ a_i a_i/4 - \varepsilon^5 a_i^+ a_i a_i^+ a_i a_i/32 + \mathrm{O}(\varepsilon^7)] \\ S_i^- &= \sqrt{2}S\hbar[\varepsilon a_i^+ - \varepsilon^3 a_i^+ a_i^+ a_i/4 - \varepsilon^5 a_i^+ a_i^+ a_i a_i^+ a_i/32 + \mathrm{O}(\varepsilon^7)]. \end{aligned} \tag{2}$$

By substituting of the above spin operator transformation into the 1D Heisenberg ferromagnet, and taking the equation of motion for operator a_i, we have

$$\mathrm{i}(\varepsilon^2/JS\hbar)\partial a_i/\partial t = F(a_i^+, a_i, a_{i+1}^+, a_{i+1}) \tag{3}$$

where F is the expression of $[a_i, H]$ in terms of $a_i, a_i^+, a_{i+1}, a_{i+1}^+$ retaining the terms of equivalent order $\mathrm{O}(\varepsilon^8)$, H is the Hamiltonian of the Heisenberg ferromagnet. Introducing Glauber's coherent state representation [4]

$$\begin{aligned} |\alpha\rangle &= \prod_i |\alpha_i\rangle \\ a_i|\alpha\rangle &= \beta_i|\alpha\rangle \end{aligned} \tag{4}$$

with normalization of the coherent state $\langle\alpha|\alpha\rangle = 1$, one can also perform the continuum limit [6–9]

$$\begin{aligned} &\beta_i \longrightarrow \beta(x,t) \\ &\beta_{i\pm1}(t) \longrightarrow \beta \pm \eta\beta_{\tilde{x}} + (1/2!)\eta^2\beta_{\tilde{x}\tilde{x}} \pm (1/3!)\eta^3\beta_{\tilde{x}\tilde{x}\tilde{x}} + (1/4!)\eta^4\beta_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}} + \mathrm{O}(\eta^5) \end{aligned} \tag{5}$$

where $\tilde{x} = x/\lambda_0$ and $\eta = a/\lambda_0$. a is a lattice constant and λ_0 is a typical wavelength of the waves (in the case of soliton excitation, λ_0 will be the soliton width) and η is the dimensionless small parameter used in the long-wave approximation. Then one can obtain an equation of motion for the 1D ferromagnet (see [8] for details):

$$\begin{aligned} \mathrm{i}\tilde{\omega}\partial\beta/\partial\tilde{t} = {} & \varepsilon^2[(\tilde{f} + 2\tau)\beta - \eta^2\beta_{\tilde{x}\tilde{x}} - \tfrac{1}{12}\eta^4\beta_{\tilde{x}\tilde{x}\tilde{x}\tilde{x}} + \mathrm{O}(\eta^6)] + \varepsilon^4\{-2\tau|\beta|^2\beta + \eta^2[-\beta|\beta_{\tilde{x}}|^2 \\ & - \tfrac{1}{2}\beta^2\beta^*_{\tilde{x}\tilde{x}} + \tfrac{1}{2}\beta^*(\beta_{\tilde{x}})^2 - \tau\beta(|\beta|^2)_{\tilde{x}\tilde{x}}] + \mathrm{O}(\eta^4)\} + \varepsilon^6[\tfrac{1}{4}|\beta|^2\beta + \mathrm{O}(\eta^2)] + \mathrm{O}(\varepsilon^8) \end{aligned} \tag{6}$$

where $\tilde{t} = \omega_0 t$ (ω_0 is the typical wave frequency), and $\tilde{f}$ and τ are dimensionless external field and anisotropy, respectively.

Taking into account the ratio of η and ε, this equation was solved [6–9]. The justification is given in our early work [6] by analogy with the theory of long waves in shallow water. However, Kapor *et al* [10] still raised a question of consistency of the approach. In the following we will clarify this point.

Let us look at a simple example with $\eta = U_1\varepsilon$ where $U_1 = \mathrm{O}(1)$. The lowest-order non-linear effects appearing in the theory are of order ε^4. Then equation (6) reduces to

$$\mathrm{i}\partial\beta/\partial t = (\mu f + 2\tau JS_c)\beta - JS_c a^2\beta_{xx} - (1/S)2JS_c\tau|\beta|^2\beta \tag{7}$$

where $S_c = \hbar S$. The soliton solution of this equation exists for $\tau > 0$, and is

$$\beta = (Sa^2/\tau)^{1/2}\nu\,\mathrm{sech}\,\nu(x - x_0 + 2JS_c a^2 kt)\mathrm{e}^{\mathrm{i}(kx-\omega t-\phi_0)} \tag{8}$$

with

$$\omega = \mu f + 2\tau J S_c + J S_c a^2(v^2 - k^2). \tag{9}$$

Kapor *et al* [10] used a 'normalization' to find a relationship between $\lambda_0 = 1/v$ and ε. They applied an *improper normalization*:

$$\frac{1}{a}\int |\beta|^2\,dx = 1 \tag{10}$$

and obtained $\lambda_0 = 2Sa/\tau$ which implies $\lambda_0 \approx \varepsilon^2$, obviously contradicting the initial assumption ($\lambda_0 \approx \varepsilon^{-1}$). Then they made a strong statement that the two-parameter theory of solitons in the spin systems [6–9] is inconsistent.

Actually, their comment is *not correct*. The coherent state we used is already normalized itself $\langle\alpha|\alpha\rangle = 1$. β is an eigenvalue of the Bose operator a acting on the coherent state $|\alpha\rangle$. *One cannot arbitrarily normalize the eigenvalue of a conherence state.* However, one can understand the relationship between η and ε in the following way.

We can take the magnetization due to a non-linear excitation as the deviation of the z projection from its maximal value:

$$M_z = \sum_i (S_c - S_i^z) = (S_c/S)\sum_i |\beta|^2. \tag{11}$$

In the continuum limit, equation (11) becomes

$$M_z = \frac{S_c}{S}\frac{1}{a}\int |\beta|^2\,dx. \tag{12}$$

By substituting equation (8) into equation (12), we obtain

$$\eta = \frac{\tau}{2}\frac{m}{S} \tag{13}$$

where $M_z = m\hbar$ is used. We can see that $\eta \approx \varepsilon$ when a physical condition, $m \approx \sqrt{S}$, is satisfied.

In conclusion, the two-parameter theory of solitons in spin systems is not inconsistent, and is useful to study a variety of non-linear excitations by varying the initial excitation conditions [6–9].

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